Mini Symposium: Singular perturbations and propagation of waves

Organizer: Didier Bresch,LMA-UBP,Clermont-Ferrand

Theory and application of initial-boundary-value problems for nonlinear wave equations.

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We will be concerned with the mathematical theory of initial-boundary-value problems for nonlinear, dispersive wave equations. The theory is somewhat more subtle than the corresponding theory for pure initial-value problems. After outlining the current state of the theory, attention will be turned to the application of this configuration in laboratory settings and in field studies connected with nearshore zone sediment transport and beach protection strategies.

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High frequency limit of the Helmholtz equation, with a source concentrating on a surface

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This talk is a review about two papers involving the above authors. We consider the high-frequency Helmholtz equation, with source term. It describes, typically, an antenna emmitting waves in the whole space. The frequency is measured by the parameter epsilon. Also, and this is the specificity of the analysis presented here, the source itself presents oscillation and concentration effects at the same scale epsilon. Using a Wigner transform approach, we analyze the asymptotic propagation of energy as epsilon goes to zero (high-frequency limit). This leads to analyzing the interactions between the oscillations carried by the source, and the oscillations induced by the Helmholtz operator, i.e. those induced by the mere propagation of waves. We prove that asymptotically the energy transport is well described by a kinetic equation with a specific source term. The latter has prescribed support both in space and frequency.

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Propagation of a laser beam in a gaz

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The propagation of a laser beam in a uranium gas in described by the Maxwell-Bloch equations. We perform a geometrical optics type limit in order to obtain the Schrödinger-Bloch system that is used for the propagation. A small parameter is introduced and an existence result on a time interval that does not depend on this parameter is proved. Some numerical simulations are given.

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The k^{-2} spectrum in rotating, forced turbulence

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Recent experiments in Swinney's lab in Texas show an interesting departure from conventional wisdom concerning spectra of rotating turbulence: there is evidence for a k^{-2} spectrum. I'll give a brief description of the experimental results and discuss some of the related mathematical results.

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Mathematical results on stability problems in fluid mechanics

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Global existence of Leray–type weak solutions for the compressible barotropic Navier–Stokes equations was proven by P.L. Lions at the beginning of the 90's. This result was the beginning of a series of studies regarding the stability of globally defined solutions, ranging from the coupling problem between weakly deformable solids evolving in a compressible viscous fluid, or the low Mach Number limit of compressible flows. Some of the filtering techniques introduced in the preceding problem have been successfully applied in the case of rapidly rotating Navier–Stokes system relevant in geophysical modeling. On the other hand, nonlinear results have been proven regarding the small time behavior of linearly unstable systems arising in fluid mechanics.

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Compressible fluids in moving domains

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The time evolution of the density rho = rho(t,x) and the velocity u = u(t,x) of a compressible viscous fluid is governed by the Navier-Stokes system $rho_t + div(rhou) = 0, (rhou)_t + div(rhou \otimes u) + gradp = muDeltau + (lambda + mu)grad(divu) + rho$ f complemented by suitable boundary conditions as the case may be. We shall discuss some recent results concerning the dependence of solutions to this problem on the spatial domain. The topics we want to address include: 1) A general existence theorem for global weak solutions on any spatial domain with no restriction on the regularity of the boundary. 2) Problems in optimal design. The dependence of the solutions on the variations of the boundary of the corresponding spatial domains. The existence of the weak solutions to certain minimization problems. 3) A general existence theorem for the weak solutions in the situation when the fluid contains moving rigid objects. The theorem takes into account possible collisions and includes rather general class of rigid objects.numerical simulations are given.

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Large time behavior for vortex evolution in the half-plane.

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In this talk we discuss the rigorous study of the time asymptotic behavior of solutions of the two dimensional Euler equations for incompressible and ideal fluids in the half-plane. Our main result is that possible asymptotic states for half-plane vortex dynamics must be a discrete superposition of states with well-defined asymptotic mean velocity. This is joint work with M. Lopes Filho and H. Nussenzveig Lopes.

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Coupling effect for Kadomtsev-Petviashvili equations

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Under the long wave approximation, the Euler equations yield, in the general $1D$ case, a system of two uncoupled KdV equations, corresponding to two counterpropagating waves. The error term is $O(\epsilon)$, where ϵ is the long wave small parameter. Under slight transverse perturbations, KdV equations must be replaced by KP equations. This can be done formally for the Euler equations, and can be proved for a model case. As for KdV, one obtains a system of uncoupled KP equations, but the error term is only $o(1)$ here. We propose to explain this difference by slight coupling effects between the two counterpropagating waves.

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On the short-wave limit of the water wave interaction equations

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This paper is devoted to the study of short-wave limit for the water wave interaction equations which arise in the study of surface waves with both gravity and capillary modes present and also in plasma physics. For the smooth solution, the limiting equation is given by the compressible Euler equation with a nonlocal pressure caused by the long wave. For weak solution, the nonlocal wave map equation is derived.

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Gravity travelling waves for two superposed fluid layers, one being of infinite depth: a new type of bifurcation

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In this talk, we study the travelling gravity waves in a system of two layers of perfect fluids, the bottom one being infinitely deep, the upper one having a finite thickness h. We assume that the flow is potential, and the dimensionless parameters are the ratio between densities $\rho = \rho_2/\rho_1$ and $\lambda = gh/c^2$. We study special values of the parameters such that $\lambda(1 - \rho)$

is near 1^-, where a bifurcation of a new type occurs: Formulating the problem as a spatial reversible dynamical system, where $U = 0$ corresponds to a uniform state (velocity c in a moving reference frame), and considering the linearized operator around 0, we show that its spectrum contains the entire real axis (essential spectrum), with in addition a double eigenvalue in 0, a pair of simple imaginary eigenvalues $\pm i\lambda$ at a distance $O(1)$ from 0, and for $\lambda(1-\rho)$ above 1, another pair of simple imaginary eigenvalues tending towards 0 as $\lambda(1-\rho) \to 1^+$. When $\lambda(1-\rho) \leq 1$ this pair disappears into the essential spectrum. The rest of the spectrum lies at a distance at least $O(1)$ from the imaginary axis. Such a bifurcation in infinite dimemsions cannot be reduced to finite dimensions using the center manifold theorem because of the lack of spectral gap of the linear operator induced by the essential spectrum lying on the whole real line. Nethertheless, we show in this talk that for $\lambda(1-\rho)$ close to 1^-, there is a family of reversible periodic solutions like in the Lyapunov-Devaney theorem (despite the resonance due to the point 0 in the spectrum). Moreover, showing that the full system can be seen as a perturbation of the Benjamin-Ono equation, coupled with a nonlinear oscillation, we also prove the existence of a family of homoclinic connections to these periodic orbits, provided that these ones are not too small.

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3D Navier-Stokes Equations with Initial Data Characterized by Uniformly Large Vorticity

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We prove existence on infinite time intervals of regular solutions to the 3D Navier-Stokes Equations for fully three-dimensional initial data in R^3 characterized by uniformly large vorticity and infinite energy; smoothness assumptions for initial data are the same as in local existence theorems. The global existence is proven using techniques of fast singular oscillating limits and the Littlewood-Paley dyadic decomposition. Infinite time regularity is obtained by bootstrapping from global regularity of the limit equations and strong convergence theorems.

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Large viscous boundary layers for noncharacterisic nonlinear hyperbolic problems

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We study linear and nonlinear stability of large-amplitude multidimensional viscous boundary layers arising through the small viscosity perturbation of a hyperbolic initial-boundary value problem with noncharacteristic boundary. The main result is to show that, provided there holds the necessary condition that all "frozen,"planar boundary layers associated with the inner layer of the profile satisfy an appropriate Evans function condition, then the linearized equations about the full profile are well-posed in L^2, with sufficiently strong estimates on the solution and its derivatives as to yield a full nonlinear stability result and thereby nonlinear continuation/validation of the formal boundary layer expansion.

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3D Euler Equations with Initial Data Characterized by Uniformly Large Vorticity

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We prove existence on arbitrary long time intervals of regular solutions to the 3D Euler Equations with initial data characterized by uniformly large vorticity; smoothness assumptions for initial data are the same as in local existence theorems. The above results are obtained in the cases of (i) bounded cylindrical domains with no flux boundary conditions, generic aspect ratios and weak alignment of the initial vorticity with the cylinder axis at t=0; (ii) 3D Euler Equations for fully three-dimensional initial data in R^3, with probability one in Vishik-Fursikov spaces of statistical solutions, characterized by uniformly large vorticity and infinite energy. The long-time existence theorems are proven using techniques of fast singular oscillating limits; the case of cylindrical geometry requires new techniques compared to periodic, infinite and/or slab domains. Algebraic geometry of resonant Poincare curves is investigated in both cases. The existence of a countable set of finite dimensional manifolds invariant under the nonlinear dynamics is demonstrated for the limit "2 1/2- dimensional" nonlinear Euler equations in generic cases.

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Refocusing for Classical Waves in Complicated Media

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We will discuss the following remarkable property of solutions of wave equations in random media (or ergodic billiards) with localized initial data. Let us record the solution at some time T, restrict it to a finite (small) domain, transform it linearly in some fashion, possibly smooth it and use the resulting signal as a new initial data for the wave equation. It turns out that the new re-propagated solution will concentrate at the original source location at the same time T for a very large class of signal processing. This is somewhat surprising since recording domain is very far from being the full space. In particular this explains the time-reversal experiments.

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Long-Time Averaging and Fast Correctors

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An alternative derivation of the averaging method for systems with two time scales will be presented, and the relationship between the correctors and the applicability of the method to large time scales will be discussed.

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Control in presence of oscillations

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In this lecture we will consider differential systems which might include, but are not limited to the discretized form of equations of the atmosphere and the ocean, such as the Lorentz equations or more involved ones. The presence of a small parameter (the Rossby number for the equations of the atmosphere) produces oscillations in the system which are well understood following recent work in the area by many authors (S. Schochet and others). We consider the control problem for these equations with the long range aim of stabilizing unstable modes by a minimal injection of energy. In this preliminary report on an ongoing work, we introduce and study the control problem, and study its behavior when the small parameter converges to zero.

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Structural analysis and boundary layer separations of 2D incompressible flows

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We present in this talk a new dynamical systems theory for 2D incompressible flows. With this theory, we derive a rigorous characterization of boundary layer separations for 2D incompressible fluids, a long standing problem in fluid dynamics.

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Approximating equations for water waves

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G. Schneider
D. Wright

I will review recent work on deriving and justifying approximating model equations for water waves. Among the equations we consider are the Korteweg-de Vries, Kawahara, and three-wave-interaction model. In each case we give rigorous estimates relating solutions of the model equation to the solutions of the full water wave equations. I will also describe some results on the form of the next order corrections to the KdV approximation.

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Stability of multidimensional viscous shocks

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We discuss a new method involving degenerate Kreiss symmetrizers for studying the stability of multidimensional viscous shocks. We discuss applications to the questions of longtime stability and also to the small viscosity limit.

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