Smooth Dynamical Systems

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Critical saddle-node bifurcations and Morse-Smale maps

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We study the dynamics of a family of diffeomorphisms just beyond a saddle-node bifurcation that destroys a stable periodic orbit, assuming that at the bifurcation parameter the orbit has a homoclinic tangency. We show that if the tangency is near critical (cubic), the family generically includes diffeomorphisms that are locally Morse-Smale for a set of parameters with positive Lebesgue density at the bifurcation parameter, while if the tangency is sufficiently far from critical, there are no Morse-Smale diffeomorphisms in the family. These results rely heavily on projecting the dynamics to circle endomorphisms. We conclude with some numerical results that indicate how common the Morse-Smale property is for near-critical circle endomorphisms.

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On the derivative formula of SRB measures

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We give the derivative formula of the Sinai-Ruelle-Bowen (SRB) measure with respect to the hyperbolic dynamical system when the foliation of the unstable manifold is smooth. Ruelle's simple formula is valid when the potential function of the SRB measure is a constant. As a consequence of this formula, we obtain the derivative formula of the entropy of the SRB measure for hyperbolic attractors.

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Dynamics of an oil spill

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We consider the evolution of a passive scalar, e.g. an oil spill, on surface of ocean (the plane), where the motion is modelled by a periodic incompressible stochastic flow. We show that for almost every realization of the random flow at time t most of the particles are at a distance of order $\sqrt{t}$ away from the origin and there is a measure zero and full Hausdorff dimension set of points, which escape to infinity at the linear rate. We study the set of points visited by the original set by time t, and show that such a set, when scaled down by the factor of t, has a limiting non random shape.

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Smooth dynamical systems

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In the early seventies Michael Shub posed the following conjecture about the topological entropy: If the universal covering space of a manifold M is homeomorphic to the Euclidean space then the topological entropy $h(f)$ of any continuous self-map $f : M \to M$ is estimated from below by the logarithm of the spectral radius $\rho(f)$ of the map $H^*(f)$ induced by f on real cohomology. This conjecture was confirmed for a torus map by Misiurewicz and Przytycki in 1977. In this work we prove the conjecture in a weaker form, in the case when M is a compact nilmanifold. An assumption we need requires that the Lefschetz number $L(f) \neq 0$, i.e. f is not homotopic to a fixed point free map. The presented method uses a notion of linearization of a self-map of nilmanifold to compare the spectral radius $\rho(f)$ with the asymptotic Nielsen number $N^\infty(f)$, and apply the Ivanov theorem. In a second theorem we drop out the assumption $L(f) \neq 0$, but we can prove then only that $h(f) > 0$ if $\rho(f) > 0$.

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